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Three-dimensional boundary layers: a report on Euromech 2

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The second European Mechanics Colloquium, on the subject of three-dimensional boundary layers, was held at Liverpool University from 4 to 7 January 1966 and was attended by thirty-eight people closely associated with current work in this field.

The meeting was opened by an introductory review by J. C. Cooke, who successfully sought to provoke discussion by emphasizing areas of apparent agreement and disagreement. The discussions which followed were based on a series of contributions by participants in the Colloquium, and covered the following topics:

(1) Laminar boundary layers.
(2) Three-dimensional perturbations of two-dimensional turbulent boundary layers.
(3) Corner and secondary flows.
(4) Boundary layers associated with flow past obstacles.
(5) Flow over delta wings.
(6) Separation.
(7) Flow over rotating surfaces.
(8) Heat and mass transfer.
(9) Miscellaneous topics.

In all these cases, the emphasis was on the three-dimensional nature of the flow, and in this report the discussions are summarized under these headings.

Introduction

The main feature of a three-dimensional boundary layer which does not appear in two dimensions is 'cross flow' or 'secondary flow' (figure 1*a*). As the main stream travels over the body the streamlines are in general curved (though the body may be plane) and this means that there is a pressure gradient normal to the streamlines (balancing 'centrifugal force') as well as a gradient along them. In accordance with the usual boundary-layer approximation, the pressure is taken to be constant across the boundary layer.

However, the fluid velocity decreases as the wall is approached and consequently the centrifugal force acting against the pressure gradient is reduced near to the wall. This causes the development of an inward component of flow (i.e. towards the centre of curvature). Hence the resultant direction of flow is now different from that of the main stream. In practice, the angle between the two directions (which is zero outside) usually increases to a maximum at the wall itself (figure 1 *b*). However, the curvature of an external streamline may change sign along its length. The effect of the consequent change in direction of the normal pressure gradient is first felt near the wall and the cross-flow first changes its sign there. The change gradually spreads outwards until ultimately the whole of the cross-flow has changed its direction. There is an intermediate position where the cross-flow is in opposite directions at different levels in the boundary layer, and the corresponding cross-flow velocity profiles have been given the name 'cross-over' profiles (figure 2). The angle between the flow directions is seldom large, perhaps not more than 15° in ordinary situations; although it may rise to a much larger value as separation is approached, the rise is fairly abrupt.

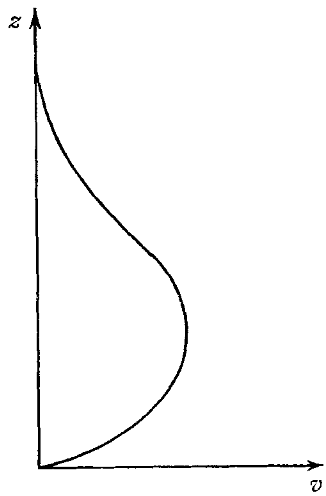

FIGURE 1(*a*). Cross-flow profile.

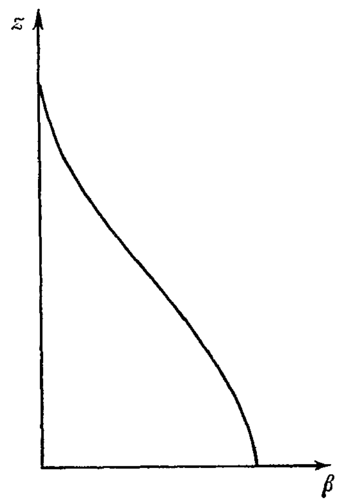

FIGURE 1(*b*). Cross-flow angle.

The varying direction of flow at different levels leads to another new feature in three-dimensional boundary layers. The effect of some change along a line normal to a point *P* on the surface is now spread over a whole growing area of the surface downstream of *P*. There is in fact a whole family of streamlines issuing in different directions at different levels from *P* and the space covered by all of these is affected by conditions at *P*. This leads to difficulties which have so far been avoided by considering quasi two-dimensional flows (swept wings, conical flows, flow over rotating axisymmetric bodies), i.e. flows with similarity in some sense. Fortunately cross-flows are usually small in turbulent flow (except perhaps near to separation) so it is to be hoped that the effects due to this feature may not be of too great importance in such a flow.

Owing to the curvature of the external streamlines and the consequent changes in the direction of the main stream it has become almost universal to use a streamline co-ordinate system in which the co-ordinate curves on the surface are the projections of the external streamlines and their orthogonal trajectories. We will denote velocity components in these directions by u and v respectively. With this system and with an assumption of small cross flow, workers have frequently made use of the 'Principle of Prevalence' due to Eichelbrenner & Oudart (1955). This amounts to assuming that v is so small that it may be ignored in the momentum equation for u, which then becomes uncoupled from the equation for v and is almost of two-dimensional form except for a term involving streamline convergence or divergence. The equation for v is also simplified by ignoring quadratic terms in v and its derivatives. This enables some progress in calculation to be made.

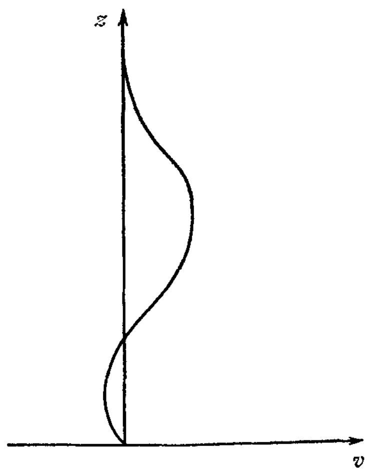

FIGURE 2. Cross-over profile.

Nevertheless, especially in turbulent flow, the adoption of a generalized form of momentum–integral procedure has usually been made. For this some form of the velocity profiles must be assumed. In turbulent flow there seems to be good reason to believe that the streamwise profile is close to a two-dimensional form, agreeing with the law-of-the-wall, and with a determinate wall skin friction, although there is some contrary evidence in the experiments of Smith (1965). It now seems clear, however, from the work of Head & Cumpsty, and of Hall, that whilst the assumption as to *profile* for u is adequate, it is not permissible to omit the v terms from the equation for u. To this extent, therefore, the Principle of Prevalence appears invalid. As regards the form to be assumed for the cross-flow, opinions differ, and of those so far used none, except that of Eichelbrenner (1955), are capable of representing cross-over profiles. Further discussion will be given later.

Another principle, known as the 'independence principle', has often been quoted in connexion with flow on infinite swept wings. In such a case it can be shown that the flow in the chordwise direction is independent of the spanwise flow; this only applies to laminar incompressible flow and may lead to considerable error if attempts are made to apply it to turbulent flow.

The notion of a 'limiting streamline' is frequently used in discussing three-dimensional boundary layers. This is the limit of a streamline as the distance from the wall tends to zero. It is also often called a 'skin friction' line since it is tangent to the direction of resultant skin friction at any point. Oil-flow patterns on the surface of a wing give an approximate picture of limiting streamlines.

Separation is another matter which requires different handling from two-dimensional flow, for which a separation point may be considered to be the point where skin friction vanishes. In three dimensions there is a separation *line*, which is usually defined to be an envelope of limiting streamlines, and along which the skin friction may remain finite.

1. Laminar boundary layers

There was only a short discussion of the problems of calculating three-dimensional laminar boundary layers. Wuest (1959) gave an account of his work on similar solutions. For special conditions he found similar solutions which depend only on one variable. The velocity profiles are then similar on the whole surface, giving 'area similarity'. For less restrictive conditions he found solutions which depend on two variables and in this case the velocity profiles are similar along lines on which one of the variables is constant ('line similarity'). The work is restricted to outer flows which follow simple power laws such as $U = Ax^n$, $V = By^n$, but the results may be useful for checking more general methods of calculation. Some cases of this and allied types have been solved by Yohner & Hansen (1958).

It was pointed out that Raetz (1957) had devised a very general method which might cover any wall-boundary conditions on temperature and suction, and any viscosity law. Lewis & Rogers (Bristol University) and Carr & Lindfield (Handley Page) are in process of programming the method, but so far have only tested it for incompressible two-dimensional flow. Also Cooke (1965) has succeeded in using a numerical method to solve problems with three dependent variables, but only two independent variables, and the method has been applied to flow about cones and to rotating axi-symmetric flows.

2. Three-dimensional perturbations of two-dimensional turbulent boundary layers

This section deals with cases where the 'Principle of Prevalence' in some form has been used. The main difficulty here concerns the type of cross-flow profile to be used.

Hall and Lewkowicz both presented experimental results which agree with the principle of prevalence. Lewkowicz's measurements of the cross-flows, carried out on a flat plate in a circular bend, also agreed well with Mager's expression

$$v/U = a(1-z/\delta)^2(u/U),$$

where $$a = \lim_{z\to 0}(v/u) \quad \text{(see figure 3).}$$

Hall's (1965) experiments were carried out in a supersonic nozzle ($M \doteqdot 1{\cdot}8$) in which there was a reversal of direction of the boundary-layer cross-flow, and cross-over profiles occurred in some places. The results do not fit Mager's (1952) model—or indeed any other model of the cross-flow which has so far been proposed.

Hornung & Joubert's (1963) experiments were also mentioned. In these an established boundary layer was deflected by an obstacle having a circular cylindrical nose with a faired tail. The arrangement was somewhat similar to that of Johnston (1960) and the authors confirmed that Johnston's triangular polar plot of the cross-flow was valid for this type of flow (see figure 4). Some analytical justification for this result was given by Perry & Joubert (1965).

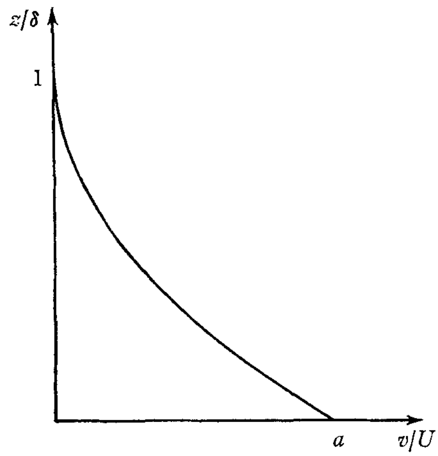

FIGURE 3. Mager profile.

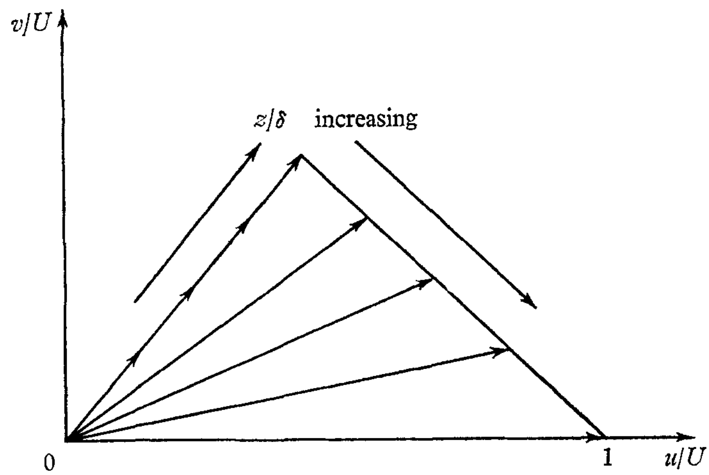

FIGURE 4. Johnston's polar plot.

Head & Cumpsty presented a method of numerical calculation based on the momentum equations (in streamwise and cross-flow directions), and using an entrainment equation similar to that used in two-dimensional flow (Head 1958). Streamwise profiles are represented by Thompson's two-dimensional family and cross-flow profiles by Mager's expression. The results of calculations for flow over the rear of an infinite swept wing were presented but no comparison with experiment has yet been made.

A somewhat similar method of calculation was presented by Lewkowicz who used Coles's (1956) velocity profile in the momentum and energy equations for the streamwise flow, together with Ludwieg–Tillmann law. He also used the

Mager expression in the cross-flow momentum integral, and obtained an exact analytical solution for the boundary layer in a curved duct with the external free stream in a state of radial equilibrium. In the discussion both Rotta and Walz emphasized their belief that the dissipation integral is dependent on the history of the boundary layer, and their suspicion of any form of expression which relates it only to local parameters.

It became clear that there is a need for a more reliable method of calculation. Cooke suggested the possibility of developing A. M. O. Smith's (unpublished) method for two-dimensional layers. This involves the use of an assumed eddy viscosity varying across the layer and then the application of laminar methods (Smith & Clutter 1963). By the principle of prevalence we may take the same formula for eddy viscosity as in two dimensions as far as the streamwise flow is concerned. Perhaps the same eddy viscosity for the cross-flow may also be taken, as Banks & Gadd (1962) assumed when dealing with ship's propellers.

3. Corner and secondary flows

The calculation of the flow along a corner has proved very difficult. It was discussed by Moore (1956) who pointed out that the effect of a corner is likely to be felt only at distances of the order of one boundary-layer thickness away from the corner and suggested that there may be a pair of vortices associated with such flows. Indeed such vortices have been observed in turbulent flows but not so far in laminar flows.

Gersten (1959) presented work on the flow in a corner, under zero and finite adverse pressure gradients. He gave results for the momentum and displacement deficits due to the corner as a function of Reynolds number, and showed that separation was reached earlier in a corner than in a two-dimensional flow. The transition point moved nearer to the leading edge as the corner was approached, but then moved downstream again very near to the corner. In his theoretical work on laminar flow Gersten developed an extension of Carrier's (1946) method. Horlock quoted the analytical work of Pearson (1958) and Louis (1958), who obtained the streamwise vorticity near the corner and the asymptotic cross-flow velocity some distance from the corner. He gave a solution in which the momentum equation was applied to show that the distance from the corner where the asymptotic cross flow was obtained was about $1\frac{1}{2}$ times the boundary-layer thickness. A more rigorous treatment of the corner flow by Rubin (1965) is to be published in this *Journal*.

Horlock and Norbury both presented results giving the velocity distribution in turbulent flow in corners. Norbury (1959) mentioned some diffuser experiments in which reversal of the streamwise vorticity occurred as secondary flows arising from shear flow curvature were replaced by secondary flows resulting from boundary-layer interference.

Fernholz described the flow in a curved wall jet bounded by plane side walls giving a passage of aspect ratio unity. Strong secondary flows were generated, producing convergence of the flow on the curved wall towards the centre line. Nevertheless, measurements of velocity profiles and skin friction (by surface Pitot tube) showed that the boundary layer on the centre line agreed well with the universal inner law. This result was also obtained for higher aspect ratios, and appeared to be generally true, including presumably the two-dimensional boundary layer on a curved wall.

Gadd presented experimental results to show that the boundary-layer interference effect at the edge of a plate extended some four boundary-layer thicknesses in from the edge.

4. Boundary layers associated with flow past obstacles

The type of flow dealt with in this section can be exemplified by consideration of the flow along a flat plate on which stands a cylinder with its generators normal to the plate. The adverse pressure gradient induced by the cylinder leads to separation from the flat plate upstream of the cylinder.

Sutton showed a remarkable series of photographs illustrating this. The separated shear layer is observed to roll up into one or more concentrated horseshoe vortices which trail downstream on either side of the cylinder. In addition the smoke showed the formation of three or more vortices ahead of the cylinder. The corresponding flows for a number of swept cylinders were also described.

Gersten (1959) reported a similar experiment in which pressure distributions on the wall were observed, and a marked depression clearly located this vortex. Gersten pointed out that an attempt to calculate the upstream separation point by two-dimensional methods had met with failure. A useful account of much of the work has been given by Steinheuer (1965). Fernholz presented the results of some measurements of skin friction distribution in the vicinity of the separating flow upstream of a nearly two-dimensional step. Preston introduced a discussion on the difficulties of observation in three-dimensional flows, and showed stereophotographs of the 'dust devil' vortices observed by Norbury at the entry to a duct located near a plane wall.

5. Flow over delta wings

When delta wings are placed at incidence in a stream the flow over the lower surface will separate from the leading edges and the separated flow forms vortex sheets which roll up into the familiar pair of coils. The flow round the outside of these sheets attaches itself to the upper surface of the wing along a pair of attachment lines which may coincide into one along the centre line of the wing. The attached fluid flows outwards and inwards from this line. That which flows outwards usually separates again forming a 'secondary separation line' (the first such line being the leading edge itself). There is then another pair of coiled vortex sheets; even tertiary separation lines have been observed.

Gregory & Love (1965) presented the results of extensive flow visualization experiments on the flow over a slender delta wing. The oil flow patterns showed separate regions of laminar and turbulent flow, lines of primary and secondary separation and attachment, and wedges of turbulence springing from the leading edges and from points on the surface. The state of the flow has been found to depend crucially on the boundary-layer behaviour near to the apex, where the flow has some resemblance to the flow past a stub circular cylinder projecting from a flat plate. In the discussion Preston commented on the smoothness of the flow reattachment and Gregory pointed out that the attaching air is not boundary-layer air, as it usually is in two-dimensional reattachment, but air that has come from the mainstream flowing round the outside of the leading edge vortices.

East presented measurements of static pressure and skin friction on a delta wing having a diamond-shaped cross-section.

6. Separation

The session on separation was introduced by Brown (1965) who pointed out that the two-dimensional criterion of separation—skin friction becoming zero—is invalid in three-dimensional flow. She discussed the consequence of regarding the separation as the envelope of the limiting streamlines, stating that for certain quasi-two-dimensional flows it is possible to determine whether or not the line is a line of singularities, but that for general three-dimensional flows this question remains unresolved. Discussion centred on the definition of separation line, it being generally agreed that it is best defined as a line dividing flow which has come from different zones, leaving open the question whether the surface flow is tangential to it.

Experimental results were presented on flows approaching separation, by Wakhaloo; on separated flows, by Schuh; and on reattaching flows, by Horton. Wakhaloo had extended Lewkowicz's experiments, showing that the Mager cross-flow model was still valid in regions of strong adverse pressure gradient, where there were large wake components in the streamwise velocity profiles. Schuh & Petterson reported on the flow in a 5° curved diffuser, followed by a straight pipe. The diffuser flow consisted of a potential core surrounded by a turbulent boundary layer in which secondary flows were induced by the curvature. Separation occurred on the inside of the bend, with a reverse flow confined to a local bubble. The bubble was closed by reattachment resulting from transverse entrainment from the non-separated boundary layer. Schuh was surprised to find that the secondary flows persisted strongly in the downstream flow, but it was reported in discussion that in other experiments (e.g. Percival 1958) similarly persistent secondary flows were observed.

Horton described some experiments on a 'swept' separation bubble, at a sweep angle of about 26°. It appears that long and short bubbles can exist just as in two-dimensional flow, and with similar criteria for their occurrence. Strong cross flows were present in the separated shear layer which became turbulent and then reattached. In this region the strong adverse pressure gradient caused rapid development of cross-flow profiles. The independence principle (i.e. independence of the streamwise flow and the cross flow) appeared to hold in the laminar part of the layer up to transition, but not thereafter.

7. Flow over rotating surfaces

The session on boundary layers associated with rotating bodies ranged over a wide field. Stewartson presented an analysis of the flow between two spheres, both rotating but with angular velocities slightly different. When the Reynolds number is high the flow outside the co-axial cylinder circumscribing the inner sphere is in solid body rotation with the angular velocity of the outer sphere, with a shear layer separating it from the inner fluid.

The shear layer is complicated in structure, having two outer layers (of thickness $\sim R_e^{-\frac{2}{7}}$ and $R^{-\frac{3}{2}}$) sandwiching an inner layer of thickness $\sim R_e^{-\frac{1}{3}}$. Inside the shear layer the fluid rotates with an angular velocity intermediate between the angular velocities of the two spheres, and the shear layer interacts with the boundary layer (of thickness $\sim R_e^{-\frac{1}{2}}$) on the outer sphere.

Gersten reported on published work by Parr (1964) in which the laminar and turbulent boundary layers on rotating axi-symmetric bodies had been measured, in accelerating and decelerating flow. This work was carried out to check the theoretical work by Schlichting (1953) and Steinheuer (1965). An interesting experimental fact is that at moderate values of the rotation parameter v_m/U, the circumferential velocity component v in the boundary layer is related to the meridional component u by the Steinheuer equation

$$v/v_m = 1 - u/U.$$

Sutton pointed out that this equation was valid for laminar flow with zero-pressure gradient, and could be obtained simply by changing from stationary to moving co-ordinates. The equation was made the basis of an approximate calculation method for laminar or turbulent flows, which gave good agreement with experiment. Gersten pointed out that the locations of transition and separation move upstream with increasing values of the rotation parameter, and remarked that the cross flows decayed only slowly on the fixed cylinder downstream of the rotating body.

Smith reported on his two-dimensional calculations of boundary-layer development up to separation on turbine and compressor blades, using several methods of calculation. The standard methods yielded different results, that of Head (1958) giving results considerably different from all the others. The method of Walz (1956) was not tried. The purpose of the calculations was to obtain optimum blade pressure distributions, but Gersten felt that true optimization was impossible because of the number of parameters involved. He also suggested that the disparity in results might be partly due to the difficulty of fixing a criterion for separation, and that calculations of losses would give more consistent results. Preston raised the question of radial boundary-layer drift but Horlock felt that in unseparated flow the evidence—both theoretical and experimental—for three-dimensional effects on turbine and compressor blades was scanty, and that two-dimensional calculations were probably good enough.

8. Heat and mass transfer

In a final session on heat transfer associated with three-dimensional flows Brun, Diep & Le Fur (1965) reported some experiments on heat and mass transfer with swept circular cylinders in a flow at $M = 2{\cdot}42$. The analogy between Nusselt and Sherwood numbers was demonstrated. Of particular interest in the mass-transfer experiments were the grooves produced in the sublimated cylinders by vortices directed along the streamlines, both in laminar and turbulent flow. In laminar flow this was due to sweep instability just before transition, and the authors believed that in the turbulent case a type of Görtler instability was involved.

Michel & Duc-Lam (1964) used the principle of prevalence, along with integral treatments of the equations of momentum and energy flux, in some calculations of skin friction and heat transfer. They predicted the turbulent skin friction on delta wings as a function of sweep and incidence angles, and calculated heat transfer coefficients on yawed cylinders. For such determinations, with small cross flows, it may not be necessary to worry about the cross-flow profiles, except near separation. Comparison of theory and experiment showed that the theory gave at least the right order of magnitude. Michel also reported on experiments in which the laminar heat transfer coefficient on the leading edges of hypersonic wings was shown to be strongly influenced by shock-wave curvature.

9. Miscellaneous topics

There were some papers which could not strictly be said to deal with three-dimensional boundary layers, but nevertheless may well have some application. Thus Schultz-Grunow (1965) dealt with laminar boundary layers on curved walls, solving Murphy's boundary-layer equations to obtain solutions to the second order in $R_e^{-\frac{1}{2}}$ for the case where the flow is similar.

Rotta dealt with the effect of curved walls on the intensity of turbulence, which is increased when the wall is concave and decreased when it is convex, the effect increasing with increasing Mach number. Corresponding extra terms were included in the energy equation, and velocity distributions were calculated on the basis of an equilibrium between the rates of turbulence production and viscous dissipation. These distributions were in qualitative agreement with experimental observations, and gave a new law of the wall, merging into the usual one near to the wall but differing in the outer part of the logarithmic region, the divergence depending on the radius of curvature (including its sign) and the 'friction Mach number' $M_\tau = u_\tau/a$.

Pichal considered the influence of mainstream turbulence (generated by oscillating vanes) on a two-dimensional turbulent boundary layer. He found a large effect on the shear stress profile; in particular it did not go to zero at the edge of the boundary layer but passed through a minimum value in that neighbourhood. Preston pointed out that the finite value of shear stress at the edge of the boundary layer was associated with a streamwise variation of turbulence.

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